

Fluctuation Theorem for a Small Engine and Magnetization Switching by Spin Torque

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We consider a reversal of the magnetic moment of a nano-magnet by the fluctuating spin-torque induced by a non-equilibrium current of electron spins. This is an example of the problem of the escape of a particle from a metastable state subjected to a fluctuating non-conservative force. The spin-torque is the non-conservative force and its fluctuations are beyond the description of the fluctuation-dissipation theorem. We estimate the joint probability distribution of work done by the spin torque and the Joule heat generated by the current, which satisfies the fluctuation theorem for a small engine. We predict a threshold voltage above which the spin-torque shot noise induces probabilistic switching events and below which such events are blocked. We adopt the theory of the full-counting statistics under the adiabatic pumping of spin angular momentum. This enables us to account for the backaction effect, which is crucial to maintain consistency with the fluctuation theorem.

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The thermodynamics of small systems, the stochastic thermodynamics [1], is of growing importance in nano-science. The key ingredient is the fluctuation theorem (FT) [1–3], which has been applied to the solid state physics recently and extends the fluctuation-dissipation theorem as well as the Onsager relations far from equilibrium (see e.g. Refs. 2–9). Recent studies suggest that the FT is also useful to analyze small engines [10–12]. In a small engine, during a short time step Δt at finite temperature T , the input heat q and the output work w fluctuate and can take positive and negative values [Fig. 1 (a)]. The FT ensures that the joint probability distribution satisfies

$$P_{R,\Delta t}(-q, -w) = P_{\Delta t}(q, w)e^{-\beta(q+w)}, \quad \beta = (k_B T)^{-1}, \quad (1)$$

where the subscript R indicates that the external driving is reversed. From Jensen's inequality, this equation reproduces the Carnot theorem, $\langle w \rangle / \langle q \rangle \leq 1$. The FT (1) is applicable even when a cycle is not defined. The work can be attributed to a non-conservative force originating from a heat flow between two baths [Fig. 1 (a)]. Let us couple the small engine to a small system. The energy variation of the small system is equal to the fluctuating work:

$$\Delta E = w. \quad (2)$$

We expect that Eqs. (1) and (2) are applicable to a wide spectrum of mesoscopic systems driven by non-conservative forces.

In the present paper, we apply this idea to the problem of the escape of a particle from a metastable state [13] subjected to a fluctuating non-conservative force. We consider the following specific setup: a nano-magnet connected to a left ferromagnetic lead (source) and a right

normal metal lead (drain) [Fig. 1 (b)]. The magnetization vector of the bulk left ferromagnetic lead \mathbf{M}_L is fixed. Let us assume that the magnetization of the nano-magnet \mathbf{M} is anti-parallel to \mathbf{M}_L . By applying a source-drain bias voltage V , spin polarized electrons are injected from the ferromagnetic lead, which exert a torque on the nano-magnet [14]. When the magnetic moment $\mathbf{M}\mathcal{V}$ (\mathcal{V} is the volume of the nano-magnet) is small, above a critical voltage V^* , \mathbf{M} is reversed and aligns parallel to \mathbf{M}_L . The spin-torque is generated by the non-equilibrium current and thus the non-conservative force. It performs the work w on the small system (the nano-magnet) and is accompanied by the Joule heat q . Since the spin angular momentum exchanged between electrons and the nano-magnet is discretized by \hbar , the spin-torque fluctuates and even under the critical voltage V^* , it can switch the magnetic moment probabilistically. The exponent Δ of the switching probability

$$P_\tau \sim e^{-\Delta}, \quad (3)$$

is well studied for equilibrium thermal fluctuations, which are Gaussian-distributed (see e.g. Refs. 15–18 and references therein). However, this is not the case for the non-equilibrium fluctuations. In current experiments [19], an MgO-insulating tunnel barrier is sandwiched between the nano-magnet and the ferromagnetic lead, which generates a Poisson-distributed shot-noise out of equilibrium [20]. Previous studies analyzing the non-equilibrium spin-torque shot noise [21–23] limited themselves to the Gaussian fluctuations. The non-Gaussian fluctuations are beyond the description of the fluctuation-dissipation theorem and, to our knowledge, have not been reliably described.

In the present paper, we determine the distribution

of non-Gaussian fluctuations by using the full-counting statistics [24] under the adiabatic pumping [25, 26], which gives the joint probability distribution consistent with the FT for a small engine (1). We evaluate the switching exponent Δ and predict another threshold voltage V_{th} under which the probabilistic switching is completely blocked. This is a result of the backaction, i.e., the adiabatic pumping of the spin angular momentum [27], as a consequence of the FT.

Langvin equation in the energy coordinate – We take the z -axis parallel to the direction of the left magnetization, $\mathbf{e}_z = (0, 0, 1) = \mathbf{M}_L/|\mathbf{M}_L|$, which is fixed [Fig. 1 (b)]. We assume the uniaxial anisotropy of the nano-magnet in the z -direction. The anisotropic energy is,

$$E = -\frac{MH_K\mathcal{V}(\mathbf{e}_z \cdot \mathbf{m})^2}{2} = -\frac{MH_K\mathcal{V}\cos^2\theta}{2}, \quad (4)$$

where $M = |\mathbf{M}|$ is the saturation magnetization and $\mathbf{m} = \mathbf{M}/M$. In the spherical coordinates, it is expressed as $\mathbf{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The anisotropic magnetic field is typically $H_K > 0$, and thus the magnetic moment tends to align with $\mathbf{m} = \mathbf{e}_z$ or $\mathbf{m} = -\mathbf{e}_z$. These 2 states are separated by the energy barrier $MH_K\mathcal{V}/2$. Because of this bistability, the setup is applicable to a memory device [19].

The dynamics of the nano-magnet is described by the stochastic Landau-Lifshitz-Gilbert equation,

$$\dot{\mathbf{m}} = -\gamma\mathbf{m} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}) + \alpha\mathbf{m} \times \dot{\mathbf{m}} - \gamma\mathbf{I}_S/(M\mathcal{V}), \quad (5)$$

where $\gamma = 2\mu_B/\hbar$ is the gyromagnetic ratio and μ_B is the Bohr magneton. The effective magnetic field is $\mathbf{H}_{\text{eff}} = -\mathcal{V}^{-1}\partial E/\partial\mathbf{M} = H_K\cos\theta\mathbf{e}_z$, and \mathbf{h} is its fluctuation induced by thermally excited magnons. It is a Gaussian white noise, i.e., $\langle h_j(t) \rangle = 0$ ($j = x, y, z$), and the correlation is instantaneous and isotropic: $\langle h_j(t)h_k(t') \rangle = 2\alpha k_B T \delta_{jk} \delta(t - t')/(\gamma M\mathcal{V})$. The Gilbert damping constant α also appears in the second term of the lhs of Eq. (5), indicating the relaxation to $\mathbf{m} = \mathbf{e}_z$ or $\mathbf{m} = -\mathbf{e}_z$. The spin-torque $\mathbf{I}_S = \mathcal{I}\mathbf{m} \times (\mathbf{e}_z \times \mathbf{m})$ aligns \mathbf{M} parallel to \mathbf{M}_L [14].

Since typically the damping and the spin torque are weak, the variation of the energy after a single precession is small [15, 17, 18]. Therefore, the magnetic moment precesses along the z axis with the frequency $\dot{\phi} = \gamma H_K \cos\theta \equiv \Omega$ along a constant energy trajectory given by Eq. (4). In the following, we will concentrate on the negative branch, $\Omega = -\sqrt{-2\gamma^2 H_K E/(M\mathcal{V})}$, i.e., $-1 \leq m_z \leq 0$. It is convenient to consider the time derivative of the energy (4) averaged over a single precession: $\overline{\dot{E}(t)} = \Omega \int_t^{t+2\pi/\Omega} dt' \dot{E}(t')/(2\pi)$. In the first order in α and \mathcal{I} , we obtain Eq. (2) for our system:

$$\overline{\dot{E}} = \overline{\mathbf{M} \cdot \partial E / \partial \mathbf{M}} = \overline{p_S} - \overline{p_\alpha}, \quad (6)$$

where $p_\alpha = \gamma M\mathcal{V}(\mathbf{m} \times \mathbf{H}_{\text{eff}}) \cdot (\alpha\mathbf{m} \times \mathbf{H}_{\text{eff}} + \mathbf{h})$ is the sum of the power dissipated by the Gilbert damping

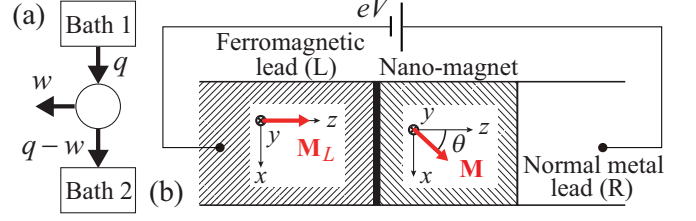


FIG. 1: (a) Schematic picture of a small engine. The input heat q and the output work w fluctuate. (b) A nano-magnet coupled to the left ferromagnetic lead and the right normal metal lead. The directions of magnetic moments of the ferromagnetic lead and the nano-magnet are $\mathbf{e}_z = (0, 0, 1)$ and $\mathbf{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.

and that generated by the thermally fluctuating magnetic field. The average is $\langle \overline{p_\alpha} \rangle = G_\alpha (\hbar\Omega)^2 \sin^2\theta$, where $G_\alpha = \pi\alpha M\mathcal{V}/(h\mu_B)$. The variance is proportional to the temperature times the average $\langle \delta p_\alpha(t) \delta p_\alpha(t') \rangle = 2k_B T \langle \overline{p_\alpha} \rangle \delta(t - t')$ ($\delta \overline{p_\alpha} \equiv \overline{p_\alpha} - \langle \overline{p_\alpha} \rangle$), which is a consequence of the fluctuation-dissipation theorem [28].

The power gain by the spin-torque is

$$\overline{p_S} = 2\mu_B \overline{\mathbf{I}_S \cdot \mathbf{H}_{\text{eff}}}/\hbar = \Omega \overline{I_{Sz}}. \quad (7)$$

For the uniaxial anisotropy case, only the z component of the spin torque is necessary $\overline{I_{Sz}} = \mathcal{I} \sin^2\theta$. Our main task is to determine the probability distribution of the fluctuating $\overline{I_{Sz}}$ to be consistent with the FT for a small engine (1).

Fluctuation theorem for non-conservative force – During a time interval Δt , which is short but sufficiently longer than the period of the precession $2\pi/\Omega$, n electrons are transmitted through the nano-magnet from left to right leads and the s electron spins flip from \uparrow to \downarrow . They are given by $n = \int_t^{t+\Delta t} dt' \overline{I(t')}/e$, where I is the charge current, and $s = \int_t^{t+\Delta t} dt' \overline{I_{Sz}(t')}/\hbar$. When the energy change is slow enough, we can calculate the joint probability distribution $P_{\Delta t}(n, s)$ using the full-counting statistics under the adiabatic pumping with the pumping frequency Ω [25, 26]. The scaled cumulant generating function (SCGF) \mathcal{F}_G is introduced as

$$\sum_{n,s} P_{\Delta t}(n, s; \Omega) e^{i\lambda n + i\chi s} \approx e^{\Delta t \mathcal{F}_G(\lambda, \chi; \Omega)}, \quad (8)$$

where λ and χ are counting fields for the numbers of transmitted electrons and flipped spins. Electrons in the left ferromagnetic lead and those in the right metal lead obey the Fermi distribution: $f_r(E) = 1/[e^{\beta(E - \mu_r)} + 1]$ ($r = L, R$). In equilibrium, the chemical potentials are at the Fermi level $\mu_L = \mu_R = E_F$. The source drain bias voltage V shifts the chemical potential of the left lead as $\mu_L = E_F + eV$.

For now, to keep the discussion simple and specific, we keep the general form of the SCGF under the adia-

batic pumping later, Eq. (20), and assume that the nano-magnet is ferromagnetic-insulating, although the current experiments use an insulator/metallic ferromagnet nano-structure [19]. The SCGF acquires the bi-directional Poisson form [29]:

$$\mathcal{F}_G(\lambda, \chi; \Omega) = \sum_{\nu, \nu' = \pm} \Gamma_{\nu\nu'}(\Omega) (e^{i\nu\lambda + i\nu'\chi} - 1) + \sum_{\pm} \Gamma_{\pm}(e^{\pm i\lambda} - 1). \quad (9)$$

The first line corresponds to the spin-flip tunneling process. The tunneling rate is

$$\Gamma_{\nu\nu'}(\Omega) = \sin^2 \theta G_{\nu\nu'} \frac{\nu eV - \nu' \hbar \Omega}{1 - e^{-\beta(\nu eV - \nu' \hbar \Omega)}}, \quad (10)$$

where $G_{++} = G_{--} = G_+$ and $G_{+-} = G_{-+} = G_-$ are spin-flip tunnel conductances. Their dimension is \hbar^{-1} and $G_{+/-}$ connects $L \uparrow / L \downarrow$ and $R \downarrow / R \uparrow$ states. The second line of Eq. (9) corresponds to the spin-preserving tunneling process.

$$\Gamma_{\nu} = [G_P \cos^2(\theta/2) + G_{AP} \sin^2(\theta/2) - \sin^2 \theta (G_+ + G_-)] (\nu eV) / (1 - e^{-\nu \beta eV}). \quad (11)$$

Similar to the free energy [30], from the derivative of the SCGF, we can calculate the charge/spin current. For example, we obtain the spin-valve expression [31],

$$\frac{\langle \bar{I} \rangle}{e} = \frac{\partial \mathcal{F}_G(\lambda, 0; 0)}{\partial (i\lambda)} \Big|_{\lambda=0} = \left(G_P \cos^2 \frac{\theta}{2} + G_{AP} \sin^2 \frac{\theta}{2} \right) eV,$$

where G_P and G_{AP} are conductances in parallel and anti-parallel alignments.

The SCGF is symmetric under the time reversal in the backward driving $\Omega \rightarrow -\Omega$. It leads the spintronic FT [8, 9]:

$$\mathcal{F}_G(\lambda, \chi; \Omega) = \mathcal{F}_{G,R}(-\lambda + i\beta eV, \chi + i\beta \hbar \Omega; -\Omega), \quad (12)$$

where the subscript R means that the magnetizations are also reversed, $\mathbf{M} \rightarrow -\mathbf{M}$ and $\mathbf{M}_L \rightarrow -\mathbf{M}_L$ (which results in $G_+ \leftrightarrow G_-$). After the inverse Fourier transform and identifying the work as $w = s\hbar\Omega$ [see Eq. (7)] and the Joule heat as $q = neV$, we obtain the FT for a small engine (1). Our SCGF (9) together with the Langevin equation in the energy coordinate (6) enables us to calculate the switching exponent consistent with the FT.

Magnetization switching – The average value of the power (7) is given by

$$\langle \overline{p_S}(\Omega) \rangle = \hbar \Omega \frac{\partial \mathcal{F}_G(0, \chi; \Omega)}{\partial (i\chi)} \Big|_{\chi=0} = \Omega I_{S_z}^{\Omega=0} - p_{\text{pump}}. \quad (13)$$

The first term is the power gain by the spin torque: $I_{S_z}^{\Omega=0} = \hbar \sin^2 \theta (G_+ - G_-) eV$. The second term is the

power dissipation by the adiabatic pumping of spin angular momentum [27]: $p_{\text{pump}} = \sin^2 \theta (\hbar \Omega)^2 (G_+ + G_-)$, which accounts for the backaction effect. We assume that initially the magnetizations are in antiparallel alignment, $m_z = \cos \theta = -1$. Then for $G_+ < G_-$, which means that the spin-flip process $L \downarrow \rightarrow R \uparrow$ is the majority process, at positive eV , there exists a frequency Ω^* at which the power gain and the power dissipation balance: $\langle \overline{p_S}(\Omega^*) \rangle = \langle \overline{p_{\alpha}}(\Omega^*) \rangle$. The condition leads, $\hbar \Omega^* = (G_+ - G_-) eV / (G_+ + G_- + G_{\alpha})$. When the magnitude of the precession frequency at $m_z = -1$, $-\Omega = \gamma H_K$ becomes smaller than $-\Omega^*$, m_z starts to increase to $m_z = 0$ and eventually reaches $m_z = 1$. The critical voltage eV^* above which the magnetization is reversed even in the absence of thermal fluctuations and spin-torque shot noise is

$$\frac{eV^*}{2\mu_B H_K} = \frac{G_+ + G_+ + G_{\alpha}}{G_- - G_+}. \quad (14)$$

Since the spin-torque shot noise is intrinsic and remains even at zero temperature, the nano-magnet switches probabilistically under eV^* . A convenient way to calculate such switching probability is the path-integral approach of the Langevin equation (6) [32]. The switching probability P_{τ} is the conditional probability to find $m_z = -1$ ($E = -MH_K \mathcal{V}/2$) at $t = 0$ and $m_z = 0$ ($E = 0$) at $t = \tau$. It is given by

$$P_{\tau} = \int \mathcal{D}\xi \int_{E(0)=-MH_K \mathcal{V}/2}^{E(\tau)=0} \mathcal{D}E e^{i\mathcal{S}}, \quad i\mathcal{S} = - \int_0^{\tau} dt \left[i\xi(t) \dot{E}(t) - \mathcal{F}_G(0, \xi(t) \hbar \Omega(t); \Omega(t)) - \mathcal{F}_{\alpha}(-\xi(t)) \right], \quad (15)$$

where we added the SCGF of Gaussian thermal noise,

$$\mathcal{F}_{\alpha}(\xi) = G_{\alpha} \sin^2 \theta (\hbar \Omega)^2 i\xi (1 + i\xi/\beta).$$

Since the number of magnetic moments in the nano-magnet $M\mathcal{V}/\mu_B$ is typically large, we utilize the optimal-path approximation. The resulting switching probability acquires the form of Eq. (3) with the switching exponent:

$$\Delta = -i\mathcal{S}^* = -\frac{M\mathcal{V}}{2\mu_B} \int_{-\gamma H_K}^{\Omega^*} d\Omega \frac{i\chi^*}{\gamma H_K}. \quad (16)$$

When the Gilbert damping is absent $\alpha = 0$, $i\chi^* = \ln[(\Gamma_{+-} + \Gamma_{--})/(\Gamma_{++} + \Gamma_{-+})]$. The solid lines in Fig. 2 are the switching exponents as a function of the bias voltage at a finite temperature and at zero temperature. We find that, at zero temperature below $eV_{\text{th}} = 2\mu_B H_K$, the exponent diverges, which means that the switching is completely blocked. This is because the spin flip process $\downarrow \rightarrow \uparrow$ is blocked: $\Gamma_{-+} + \Gamma_{--} = 0$. At finite temperature, this divergence disappears and at $eV = 0$, we obtain the Arrhenius law: $\Delta = MH_K \mathcal{V} / (2k_B T)$. The inset shows results at a finite α . We see that the divergence remains.

Close to the critical voltage, we approximate $i\chi^* \approx (\hbar\Omega - \hbar\Omega^*)/(k_B T_{\text{eff}})$ and obtain the Arrhenius-like form

$$\Delta = \frac{MH_K\mathcal{V}}{2k_B T_{\text{eff}}} \left(\frac{V^* - V}{V^*} \right)^2, \quad (17)$$

which quadratically depends on the distance from the critical voltage. The effective temperature,

$$T_{\text{eff}} = \frac{\sum_{\pm} G_{\mp}(eV \pm \hbar\Omega^*) \coth \frac{eV \pm \hbar\Omega^*}{2k_B T} + 2k_B T G_{\alpha}}{2k_B (G_+ + G_- + G_{\alpha})},$$

is reduced to the real temperature $T_{\text{eff}} \approx T$ for high temperatures, $eV, eV_{\text{th}} \ll k_B T$. Then Eq. (17) reproduces the previous result [16, 18]. At zero temperature and $G_{\alpha} = 0$, $T_{\text{eff}} \approx 2G_+ G_- eV / (G_+ + G_-)^2$, which is proportional to the bias voltage [22], indicating that the spin-torque shot noise is the dominant source of fluctuations around the critical voltage. The dashed lines in Fig. 2 show Eq. (17). They fit well for finite temperature or around the critical voltage.

When the volume becomes very small, i.e., $\mathcal{V} \sim \mu_B/M$, we have to go beyond the optimal path approximation [34, 35]. In such cases, the time scales of the source of Gaussian noise and that of Poisson noise should be treated carefully [35].

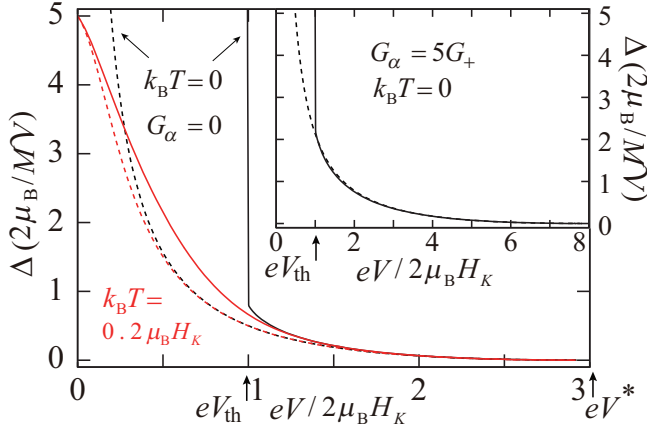


FIG. 2: The bias-voltage dependence of the switching exponent Δ for $\alpha = 0$ and $G_- = 2G_+$. The two solid lines are for different temperatures $k_B T / (2\mu_B H_K) = 0$ and 0.1 . The inset shows a plot for finite $G_{\alpha} = 5G_+$. The dashed lines indicate an Arrhenius-like law, Eq. (17). The critical voltages are $eV^*/(2\mu_B H_K) = 3$ and 8 for $G_{\alpha} = 0$ and $5G_+$.

Full-counting statistics under the adiabatic pumping – An electron transferred through the nano-magnet is affected by its precession motion. This scattering process is described by a time-dependent S -matrix:

$$\mathbf{S}(\theta, \phi(t)) = e^{-i\phi(t)\sigma_z/2} \mathbf{S}(\theta) e^{i\phi(t)\sigma_z/2} \quad (18)$$

$$\mathbf{S}(\theta) = \begin{pmatrix} \mathbf{r}(\theta) & \mathbf{t}'(\theta) \\ \mathbf{t}(\theta) & \mathbf{r}'(\theta) \end{pmatrix}, \quad (19)$$

where the Pauli matrix σ_z acts in the spin space and $\phi(t) \approx \Omega t + \phi(0)$. \mathbf{r} and \mathbf{t} (\mathbf{r}' and \mathbf{t}') are 2×2 matrix of the spin-dependent reflection amplitudes and that of the spin-dependent transmission amplitudes for an incoming wave from the left (right) lead. For example, the element $t_{\sigma'\sigma}$ describes an electron transmission from the spin σ state in the left lead to the spin σ' state in the right lead.

The SCGF (8) is expressed by using the S -matrix as [25, 26],

$$\mathcal{F}_G(\lambda, \chi; \Omega) = \sum_{\ell} \int \frac{dE}{h} \ln \det \left[\mathbf{1} - \mathbf{f}(E) \left(\mathbf{1} - e^{i\boldsymbol{\lambda} + i\chi\sigma_z/2} \mathbf{S}^{\ell}(\theta; E)^{\dagger} e^{-i\boldsymbol{\lambda} - i\chi\sigma_z/2} \mathbf{S}^{\ell}(\theta; E) \right) \right], \quad (20)$$

where $\mathbf{S}^{\ell}(E)$ is the S -matrix for the ℓ -th transverse channel. The counting field matrix $\boldsymbol{\lambda} = \text{diag}(\lambda, \lambda, 0, 0)$ counts the number of electrons flowing out of the left lead. The precession motion effectively splits the \uparrow -spin and \downarrow -spin chemical potentials of the 2 leads after the gauge transform, $\mathbf{f}(E) = \text{diag}(f_L(E + \hbar\Omega/2), f_L(E - \hbar\Omega/2), f_R(E + \hbar\Omega/2), f_R(E - \hbar\Omega/2))$. The spin-splitting of the chemical potentials is a result of the backaction, which is crucial to be consistent with the FT [33]. It also blocks the spin-flip tunneling process $\downarrow \rightarrow \uparrow$ under the threshold voltage.

Although we considered a simple model, it is also possible to calculate the S -matrix using a realistic model. Then, from Eq. (20), we obtain Eq. (13) expressed with general $I_{S_z=0}^{\Omega=0}$ and p_{pump} ,

$$I_{S_z=0}^{\Omega=0} = \frac{eV}{4\pi} \sum_{\ell, \sigma=\uparrow, \downarrow} (|r_{\downarrow\sigma}^{\ell}(\theta, \phi; E_F)|^2 + |t_{\downarrow\sigma}^{\ell}(\theta, \phi; E_F)|^2 - |r_{\uparrow\sigma}^{\ell}(\theta, \phi; E_F)|^2 - |t_{\uparrow\sigma}^{\ell}(\theta, \phi; E_F)|^2),$$

$$p_{\text{pump}} = \frac{\hbar\Omega^2}{4\pi} \sum_{\ell} \text{tr}(\partial_{\phi} \mathbf{S}^{\ell}(\theta, \phi; E_F) \partial_{\phi} \mathbf{S}^{\ell}(\theta, \phi; E_F)^{\dagger}),$$

in the leading order of eV and Ω . It is straightforward to take the channel mixing scattering into account. Our p_{pump} reproduces Ref. 27.

Summary – We demonstrate the switching probability driven by fluctuating non-conservative spin-torque. The theory of the full-counting statistics under the adiabatic pumping enables us to account for the backaction effect and to obtain a distribution of the fluctuating spin-torque consistent with the fluctuation theorem for a small engine. We find the threshold voltage $eV_{\text{th}} = 2\mu_B H_K$, above which the spin-torque shot noise causes the probabilistic switching. Under the threshold the spin-flip tunneling process is blocked because of the backaction and thus the probabilistic switching is suppressed.

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SUPPLEMENTAL MATERIAL

Technical details of derivations of a scattering matrix, a scaled cumulant generating function and a switching exponent.

Scattering matrix and the scaled cumulant generating function

We derive the S -matrix of the ferromagnet/ferromagnetic insulator/normal metal structure. We take the z -axis perpendicular to the interface and assume translational invariance in the x and y directions. The Schrödinger equation is

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + U(z)\right)\psi(x, y, z) = E\psi(x, y, z), \quad U(z) = \begin{cases} \mu_B H_{mL} \sigma_z / 2 & (z < 0) \\ U_0 + \mu_B H_m \mathbf{m} \cdot \vec{\sigma} & (0 \leq z < d) \\ 0 & (d \leq z) \end{cases}, \quad (21)$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrix vector. The thickness of the ferromagnetic insulator is d and $U_0 > 0$ is the potential barrier height. The molecular (exchange) fields in the ferromagnetic lead and in the ferromagnetic insulator are H_{mL} and H_m , respectively. The wave function is written as $\psi(x, y, z) = 2 \sin(\pi \ell_x x / L) \sin(\pi \ell_y y / L) \psi(z) / L$ where the contact area is $0 \leq x, y \leq L$ (ℓ_y and ℓ_z are non-negative integers). In the z direction, the Schrödinger equation reads

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U(z)\right)\psi_\ell(z) = E_\ell \psi_\ell(z), \quad E_\ell = E - \frac{\hbar^2 \pi^2}{2mL^2}(\ell_y^2 + \ell_z^2), \quad (22)$$

where we introduced the channel index $\ell = (\ell_x, \ell_y)$. The wave number of an electron with the energy E is $k_\sigma = \sqrt{2m(E_\ell - \sigma \mu_B H_{mL})} / \hbar$ in the ferromagnetic lead ($z < 0$), $ik_\sigma = \sqrt{2m(E_\ell - U_0 - \sigma \mu_B H_m / 2)} / \hbar$ in the ferromagnetic

insulator ($0 < z < d$) and $k = \sqrt{2mE_\ell}/\hbar$ in the normal metal lead ($d < z$). The S -matrix in the leading order of $e^{-\kappa_\sigma d}$ is calculated as

$$\mathbf{S}(\theta) = \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}' \end{pmatrix} \begin{pmatrix} -\mathbf{1} - (i\mathbf{A} + \boldsymbol{\tau}^\dagger \boldsymbol{\tau}/2) & \boldsymbol{\tau}^\dagger \\ \boldsymbol{\tau} & \mathbf{1} + (i\mathbf{A} + \boldsymbol{\tau} \boldsymbol{\tau}^\dagger/2) \end{pmatrix} \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}' \end{pmatrix}. \quad (23)$$

where an Hermite matrix $\mathbf{A}^\dagger = \mathbf{A}$ is not relevant for our model. Further, we neglect H_m except when it appears in the exponent of $e^{-\kappa_\sigma d}$. Then we obtain the following 2×2 matrix of the spin-dependent transmission amplitude:

$$\boldsymbol{\tau} = \frac{1}{2} \begin{pmatrix} \tau_{\uparrow\uparrow} + \tau_{\downarrow\uparrow} + (\tau_{\uparrow\uparrow} - \tau_{\downarrow\uparrow}) \cos \theta & (\tau_{\uparrow\downarrow} - \tau_{\downarrow\downarrow}) \sin \theta \\ (\tau_{\uparrow\uparrow} - \tau_{\downarrow\uparrow}) \sin \theta & \tau_{\uparrow\downarrow} + \tau_{\downarrow\downarrow} + (\tau_{\uparrow\downarrow} - \tau_{\downarrow\downarrow}) \cos \theta \end{pmatrix}, \quad \tau_{\sigma\sigma'} = 4e^{-\kappa_\sigma d} \sqrt{\frac{\kappa_0 k}{\kappa_0^2 + k^2} \frac{\kappa_0 k_{\sigma'}}{\kappa_0^2 + k_{\sigma'}^2}}. \quad (24)$$

2×2 sub-matrices \mathbf{P} , \mathbf{P}' become diagonal and (σ, σ') component of \mathbf{P}^2 and \mathbf{P}'^2 are $(\kappa_0 + ik_\sigma)/(\kappa_0 - ik_\sigma)$ and $-i(\kappa_0 + ik)/(\kappa_0 - ik)$, where $\kappa_0 = \sqrt{2m(U_0 - E_\ell)}/\hbar$.

We insert the S -matrix (23) into Eq. (20) in the main text:

$$\mathcal{F}_G(\lambda, \chi; \hbar\Omega) = \rho_\parallel \int dE_\parallel \int \frac{dE}{\hbar} \ln \det \left[\mathbf{1} + \mathbf{f}(E) \left(e^{i\boldsymbol{\lambda} + i\chi\sigma_z/2} \mathbf{S}(E - E_\parallel, \theta)^\dagger e^{-i\boldsymbol{\lambda} - i\chi\sigma_z/2} \mathbf{S}(E - E_\parallel, \theta) - \mathbf{1} \right) \right], \quad (25)$$

where $\rho_\parallel = 2\pi m L^2/\hbar^2$ is the DOS of the transverse channel. Since the energy dependence of $\tau_{\sigma\sigma'}$ is small around the Fermi energy E_F , it is possible to approximate $\tau_{\sigma\sigma'}(E - E_\parallel) \approx \tau_{\sigma\sigma'}(E_F) \exp(-E_\parallel/(2\delta))$, where $\delta^{-1} = 2d \partial \kappa_0(E_\ell = E_F)/\partial E_\ell$. After performing the integral and expanding up to the leading order in $e^{-\kappa_\sigma d}$, we obtain Eq. (9) in the main text. The conductances are

$$G_+ = \frac{1}{\hbar} \rho_\parallel \delta |\tau_{\downarrow\downarrow}(E_F) - \tau_{\uparrow\downarrow}(E_F)|^2, \quad (26)$$

$$G_- = \frac{1}{\hbar} \rho_\parallel \delta |\tau_{\uparrow\uparrow}(E_F) - \tau_{\downarrow\uparrow}(E_F)|^2, \quad (27)$$

$$G_P = \frac{1}{\hbar} \rho_\parallel \delta (|\tau_{\uparrow\uparrow}(E_F)|^2 + |\tau_{\downarrow\downarrow}(E_F)|^2), \quad (28)$$

$$G_{AP} = \frac{1}{\hbar} \rho_\parallel \delta (|\tau_{\uparrow\downarrow}(E_F)|^2 + |\tau_{\downarrow\uparrow}(E_F)|^2). \quad (29)$$

The reversal of the magnetic moments $\mathbf{M} \rightarrow -\mathbf{M}$ and $\mathbf{M}_L \rightarrow -\mathbf{M}_L$, which corresponds to $H_m \rightarrow -H_m$ and $H_{mL} \rightarrow -H_{mL}$, changes the tunneling amplitude to $\tau_{\sigma\sigma'} \rightarrow \tau_{\sigma'\sigma}$ and thus the conductances to $G_+ \leftrightarrow G_-$.

Switching exponent

We analyze the Langevin equation (6) in the main text by exploiting the Martin-Siggia-Rose approach (see Section 4 in Ref. 1). We first discretize time τ into $N = \tau/\Delta t$ steps. For now, we neglect the equilibrium power dissipation p_α . The variation of the energy during a short time step from $t_j = \Delta t j$ to t_{j+1} is

$$E_{j+1} - E_j \approx \hbar \Omega_j s_j, \quad s_j = \int_{t_j}^{t_{j+1}} dt \overline{I_{Sz}(t)}, \quad (30)$$

where $E_j = E(t_j)$ and $\Omega_j = \Omega((E_{j+1} + E_j)/2)$. The stochastic variable s_j is distributed according to the joint probability distribution Eq. (8) described in the main text. The conditional joint probability to find E_j at time t_j and E_{j+1} at t_{j+1} accompanied by n_j electron transmission is given by

$$P_{\Delta t}(n_j, E_{j+1}|E_j) = \int d\epsilon_{Sj} \delta(E_{j+1} - E_j - \epsilon_{Sj}) \sum_{s,n} P_{\Delta t}(n, s; \Omega_j) \delta(\epsilon_{Sj} - \hbar \Omega_j s) \delta_{n_j, n} \quad (31)$$

$$= \int_{-\pi}^{\pi} \frac{d\lambda_j}{2\pi} \int \frac{d\xi_j}{2\pi} e^{-i\lambda_j n_j - i\xi_j (E_{j+1} - E_j) + \mathcal{F}_G(\lambda_j, \hbar \Omega_j \xi_j; \Omega_j) \Delta t}. \quad (32)$$

Then the conditional joint probability to find $E(0)$ at $t = 0$ and $E(\tau)$ at τ accompanied by n electron transmission is calculated by accumulating joint probabilities for short time steps as follows:

$$\begin{aligned} P_\tau(n, E(\tau)|E(0)) &= \sum_{n_0, \dots, n_{N-1}} \int dE_1 \cdots dE_{N-1} P_{\Delta t}(n_{N-1}, E_N|E_{N-1}) \cdots P_{\Delta t}(n_0, E_1|E_0) \delta_{n, \sum_{j=0}^{N-1} n_j} \\ &= \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} \int \frac{d\xi_0}{2\pi} \cdots \frac{d\xi_{N-1}}{2\pi} \int dE_1 \cdots dE_{N-1} e^{\sum_{j=0}^{N-1} [-i\xi_j (E_{j+1} - E_j) + \mathcal{F}_G(\lambda, \xi_j \hbar \Omega_j; \Omega_j) \Delta t] - i\lambda n}. \end{aligned} \quad (33)$$

We can prove the detailed FT by Jarzynski [2] based on the FT (12) in the main text along the same line of the proof in Ref. 3,

$$P_\tau(n, E(\tau)|E(0))/P_{R,\tau}(-n, E(0)|E(\tau)) = e^{\beta[n eV - E(\tau) + E(0)]}, \quad (34)$$

In order to account for the equilibrium power dissipation, we can just replace \mathcal{F}_G with $\mathcal{F}_G(\lambda, \xi\hbar\Omega; \Omega) + \mathcal{F}_\alpha(-\xi)$. Further, for calculating the switching rate, we can sum over n , $P_\tau(E(\tau)|E(0)) \equiv \sum_n P_\tau(n, E(\tau)|E(0))$. Then in the continuous limit, $\Delta t \rightarrow 0$, we obtain the path-integral form Eq. (15) in the main text.

Since $E, G_\alpha \propto \mathcal{V} = L^2 d$ and $\mathcal{F}_G \propto L^2/d$, for a modestly large nano-magnet, it is possible to perform the optimal path approximation [1, 4–6]. Namely, from the variational principle, we derive the “canonical equation of motion”:

$$\dot{E} = \frac{\partial \mathcal{F}}{\partial(i\xi)}, \quad i\dot{\xi} = -\frac{\partial \mathcal{F}}{\partial E}. \quad (35)$$

The “momenta” $i\xi$ measures the strength of the fluctuations. $i\xi = 0$ corresponds to the noiseless case, which is always an optimal path. The equation of motion possesses the integral of motion, which is the “energy,” \mathcal{F} . Since the normalization condition ensures $\mathcal{F}(\xi = 0; \Omega) = 0$, the optimal paths always satisfy $\mathcal{F}(\xi; \Omega) = 0$.

We are interested in an optimal path that starts from $(E, i\xi) = (-MH_K\mathcal{V}/2, 0)$ and reaches $(E, i\xi) = (0, 0)$. For $\alpha = 0$, we find 4 simple solutions satisfying $\mathcal{F}(\xi; \Omega) = 0$:

$$\hbar\Omega(E) = \pm 2\mu_B H_K, \quad i\hbar\Omega(E)\xi^* = \ln \frac{\Gamma_{+-} + \Gamma_{--}}{\Gamma_{++} + \Gamma_{-+}}, \quad i\xi^* = 0. \quad (36)$$

Figure 3 (a) shows the optimal paths. The horizontal axis is $\Omega = -\sqrt{-2\gamma^2 H_K E/(M\mathcal{V})}$ and thus $E = -MH_K\mathcal{V}/2$ and $E = 0$ correspond to $\hbar\Omega = -2\mu_B H_K$ and $\hbar\Omega = 0$, respectively. Arrows indicate the directions of motion determined from Eq. (35). The initial state is at M, i.e., $(\hbar\Omega, i\xi\hbar\Omega) = (-2\mu_B H_K, 0)$, and the final state is at T, i.e., $(\hbar\Omega, i\xi\hbar\Omega) = (0, 0)$. The optimal path is $M \rightarrow M' \rightarrow U \rightarrow T$, where the intermediate state U is $(\hbar\Omega, i\xi\hbar\Omega) = (\hbar\Omega^*, 0)$. The action along this path is calculated as

$$i\mathcal{S}^* = - \int_{-MH_K\mathcal{V}/2}^{E(\Omega^*)} dE (i\xi^*) = \frac{M\mathcal{V}}{2\mu_B\gamma H_K} \int_{-2\mu_B H_K}^{\Omega^*} d\Omega i\xi^* \hbar\Omega \equiv -\Delta. \quad (37)$$

The integral $\int d\Omega i\xi^* \hbar\Omega$ gives the area of the shaded region in Fig. 3 (a). This equation leads to Eq. (16) in the main text and the switching probability up to the single instanton contribution, $P_\tau(E(\tau) = 0|E(0) = -MH_K\mathcal{V}/2) \approx e^{-\Delta}$.

At zero temperature, the integral (37) can be performed easily. For $eV_{\text{th}} = 2\mu_B H_K < eV < eV^*$, we obtain

$$i\mathcal{S}^* = \frac{M\mathcal{V}}{2\mu_B} \left\{ \ln \frac{G_-(eV - 2\mu_B H_K)}{G_+(eV + 2\mu_B H_K)} + \frac{eV}{2\mu_B H_K} \ln \frac{4G_+G_-(eV)^2}{(G_+ + G_-)^2[(eV)^2 - (2\mu_B H_K)^2]} \right\}. \quad (38)$$

For $eV < eV_{\text{th}}$, it diverges to $i\mathcal{S}^* = -\infty$, which means that the switching is completely blocked. Figure 3 (b) shows the optimal path at $eV = eV_{\text{th}}$. M' approaches $(\hbar\Omega, i\xi\hbar\Omega) = (0, -\infty)$ in the limit of zero temperature, and the area of the shaded region diverges. For $G_\alpha \neq 0$, the optimal path is modified and we determine it numerically.

With increasing bias voltage, the shaded area decreases and eventually M' and U meet at M [Fig. 3 (c)]. The exponent and the switching probability become $i\mathcal{S}^* = 0$ and $P_\tau \approx 1$. This critical condition is achieved at $\hbar\Omega^* = -2\mu_B H_K$, which is equivalent to the balance condition $\langle p_S(\Omega^*) \rangle = \langle p_\alpha(\Omega^*) \rangle$. Around the critical point (for $G_\alpha \neq 0$), we can expand ξ^* around $\Omega = \Omega^*$ and $\xi = 0$, up to the lowest order as

$$i\xi^* \hbar\Omega \approx \frac{\hbar\Omega - \hbar\Omega^*}{k_B T_{\text{eff}}}.$$

By plugging this expression into Eq. (37), we obtain Eq. (17) in the main text.

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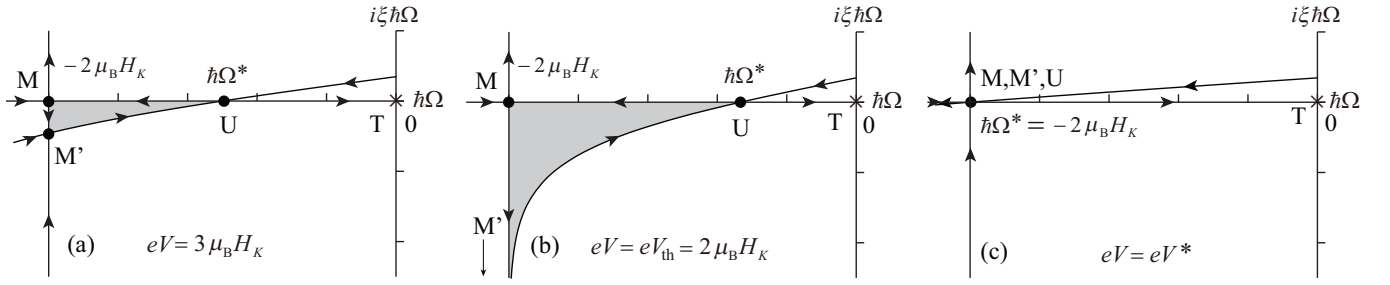


FIG. 3: The optimal paths (a) for $eV = 3\mu_B H_K$, (b) for the threshold voltage $eV = eV_{\text{th}} = 2\mu_B H_K$ and (c) for the critical voltage $eV = eV^* = 6\mu_B H_K$. The parameters are as follows: $G_- = 2G_+$, $G_\alpha = 0$ and $k_B T = 0$.